9th Class 2015

Math (Science)	Group-I	Paper-
Time: 2.10 Hours	(Subjective Type)	Max. Marks: 6

(Part-I)

2. Write short answers to any Six (6) questions: 12

(i) Find |C| : $C = \begin{bmatrix} 3 & 2 \\ 3 & 2 \end{bmatrix}$

Ans

$$C = \begin{bmatrix} 3 & 2 \\ 3 & 2 \end{bmatrix}$$

$$|C| = \begin{vmatrix} 3 & 2 \\ 3 & 2 \end{vmatrix}$$

$$|C| = (3)(2) - 3(2)$$

$$|C| = 6 - 6$$

$$|C| = 0$$

(ii) Find the product: $\begin{bmatrix} 1 & 2 \\ -3 & 0 \\ 6 & -1 \end{bmatrix} \begin{bmatrix} 4 & 5 \\ 0 & -4 \end{bmatrix}$

Ans $\begin{bmatrix} 1 & 2 \\ -3 & 0 \\ 6 & -1 \end{bmatrix} \begin{bmatrix} 4 & 5 \\ 0 & -4 \end{bmatrix}$

1^{st} matrix has (3×2) order, while 2^{nd} matrix has (2 × 2) order. As number of columns of 1^{st} matrix is equal to number of rows of 2^{nd} matrix, thus product of above matrices will exist. So,

$$\begin{bmatrix} 1 & 2 \\ -3 & 0 \\ 6 & -1 \end{bmatrix} \begin{bmatrix} 4 & 5 \\ 0 & -4 \end{bmatrix}$$

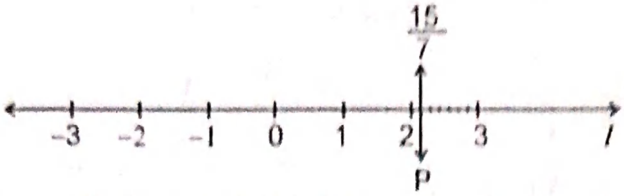
$$= \begin{bmatrix} 1(4) + 2(0) & 1(5) + 2(-4) \\ -3(4) + 0(0) & -3(5) + 0(-4) \\ 6(4) + (-1)(0) & 6(5) + (-1)(-4) \end{bmatrix}$$

$$= \begin{bmatrix} 4 + 0 & 5 - 8 \\ -12 + 0 & -15 + 0 \\ 24 + 0 & 30 + 4 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & -3 \\ -12 & -15 \\ 24 & 34 \end{bmatrix}$$

(iii) Represent the number $\frac{15}{7}$ on the number line.

Ans $\frac{15}{7} = 2\frac{1}{7}$: It lies between 2 and 3.



Divide the distance between 2 and 3 into seven equal parts. The point P represents the number $\frac{15}{7} = 2\frac{1}{7}$.

(iv) Write the real and imaginary part of the number: 2 + 0i:

Ans

Real part = 2

Imaginary part = 0

(v) Write into sum or difference: $\log \frac{(22)^{1/3}}{5^3}$

Ans

$$\log \frac{(22)^{1/3}}{5^3}$$
$$= \log (22)^{1/3} - \log (5)^3$$
$$= \frac{1}{3} \log (22) - 3 \log (5)$$

(vi) Write in the form of a single logarithm:
log 25 – 2 log 3

Ans

$$\log 25 - 2 \log 3$$
$$= \log 25 - \log 3^2$$
$$= \frac{\log 25}{\log 3^2} = \log \frac{25}{3^2}$$

(vii) Reduce the algebraic fraction to their lowest form:

$$\frac{lx + mx - ly - my}{3x^2 - 3y^2}$$

Ans $$\frac{lx + mx - ly - my}{3x^2 - 3y^2} = \frac{x(l + m) - y(l + m)}{3(x^2 - y^2)}$$

$$= \frac{(l+m)(x-y)}{3(x+y)(x-y)} \quad \text{(Factorizing)}$$

$$= \frac{l+m}{3(x+y)} \quad \text{(Canceling common factors)}$$

Which is in the lowest form.

(viii) Evaluate $\frac{x^2y - 2z}{xz}$ for $x = 3, y = -1, z = -2$

Ans By putting the values $x = 3$, $y = -1$, $z = -2$ in the expression $\frac{x^2y - 2z}{xz}$:

$$= \frac{(3)^2(-1) - (2)(-2)}{(3)(-2)}$$

$$= \frac{9(-1) + 4}{-6}$$

$$= \frac{-9 + 4}{-6}$$

$$= \frac{-5}{-6}$$

$$= \frac{5}{6}$$

(ix) Define remainder theorem.

Ans If a polynomial p(x) is divided by a linear divisor (x - a) then the remainder is p(a).

3. Write short answers to any Six (6) questions:

(i) Find square root of the following $4x^2 - 12x + 9$ by factorization

Ans Factorization of the expression $4x^2 - 12x + 9$ is of the type: $(a)^2 - 2(a)(b) + (b)^2 = (a - b)^2$

So, $4x^2 - 12x + 9$

$$= (2x)^2 - 2(2x)(3) + (3)^2$$

$$= (2x - 3)^2$$

Taking square root

$$= \pm(2x - 3)$$

(ii) Solve it: $\sqrt[3]{2x - 4} - 2 = 0$

Ans

$$\sqrt[3]{2x - 4} = 2$$

$$(2x - 4)^{1/3} = 2$$

By taking cube both sides,

$$[(2x - 4)^{1/3}]^3 = (2)^3$$
$$2x - 4 = 8$$
$$2x = 8 + 4$$
$$2x = 12$$
$$x = \frac{12}{2}$$
$$\boxed{x = 6}$$

(iii) Find the solution set of the equation: $\left|\frac{x + 5}{2 - x}\right| = 6$

Ans

$$\left|\frac{x + 5}{2 - x}\right| = 6$$
$$\pm\left(\frac{x + 5}{2 - x}\right) = 6$$

$$\frac{x + 5}{2 - x} = 6$$
$$x + 5 = 6(2 - x)$$
$$x + 5 = 12 - 6x$$
$$x + 6x = 12 - 5$$
$$7x = 7$$
$$x = \frac{7}{7}$$
$$\boxed{x = 1}$$

$$-\left(\frac{x + 5}{2 - x}\right) = 6$$
$$-(x + 5) = 6(2 - x)$$
$$-x - 5 = 12 - 6x$$
$$-x + 6x = 12 + 5$$
$$5x = 17$$
$$\Rightarrow \boxed{x = \frac{17}{5}}$$

So, solution set $= \left\{1, \frac{17}{5}\right\}$.

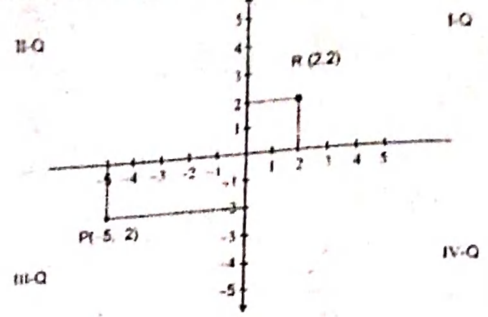
(iv) Define Cartesian Plane.

Ans The Cartesian plane establishes (one-to-one) correspondence between the set of ordered pairs $R \times R = \{(x, y) \mid x, y \in R\}$ and the points of the Cartesian plane.

(v) Draw in which quadrant points lie P(–5, –2), R(2, 2).

Ans

Ordered Pair	Quadrant
P(–5, –2)	III – Q
R(2, 2)	I – Q



(vi) Define equilateral triangle.

Ans If the lengths of all the three sides of a triangle ar[e] same, then the triangle is called an equilateral triangle.

(vii) Find the distance between points

$A(-4, \sqrt{2}), B(-4, -3)$

Ans Formula for distance between two points:

$$d = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$$

Here, from question:

$$x_1 = -4 \quad , \quad y_1 = \sqrt{2}$$
$$x_2 = -4 \quad , \quad y_2 = -3$$

By putting, we get:

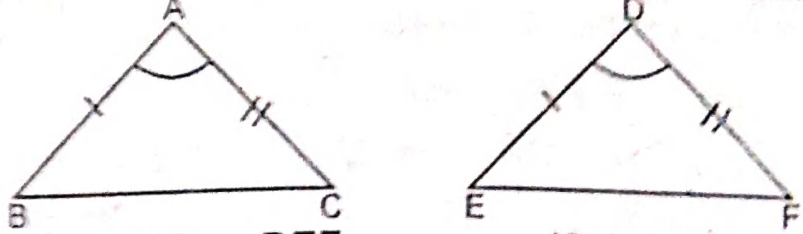
$$d = \sqrt{[(-4) - (-4)]^2 + [(-3) - \sqrt{2}]^2}$$
$$d = \sqrt{(-4 + 4)^2 + (-3 - \sqrt{2})^2}$$
$$d = \sqrt{0 + [(-1)(3 + \sqrt{2})]^2}$$
$$d = \sqrt{(-1)^2(3 + \sqrt{2})^2}$$
$$d = \sqrt{(3 + \sqrt{2})^2}$$
$$d = 3 + \sqrt{2}$$

(viii) What do you mean by SAS ≅ SAS?

Ans In any correspondence of two triangles, if two sides and their included angle of one triangle are congruent t[o] the corresponding two sides and their included angle o[f] the other, then the triangles are congruent.

In $\Delta ABC \longleftrightarrow \Delta DEF$, shown in the following figures,

$$\text{if} \begin{cases} \overline{AB} \cong \overline{DE} \\ \angle A \cong \angle D \\ \overline{AC} \cong \overline{DF} \end{cases}$$



then $\Delta ABC \cong \Delta DEF$ (S.A.S Postulate)

(ix) **Define parallelogram.**

Ans A figure formed by four non-collinear points in the plane is called parallelogram. It characteristics are as under:

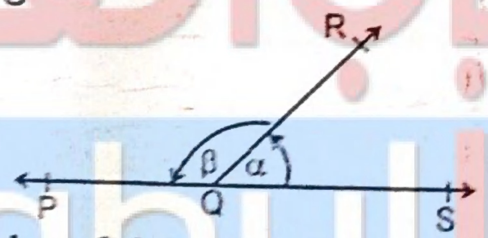
1. Its equal opposite sides are of equal measure.
2. Its opposite sides are parallel.
3. Measure of none of the angle is 90°.

4. Write short answers to any Six (6) questions: 12

(i) **Define supplementary angles. Give an example.**

Ans Supplementary angles are two angles whose sum is 180°. If the sum of two angles is 180°, then each angle is called the supplement of the other.

For example, in the following figure, $\angle\alpha$ and $\angle\beta$ are supplementary angles.



(ii) **Can a triangle of lengths 2 cm, 4 cm and 7 cm be formed? Give reason.**

Ans 2 cm, 4 cm and 7 cm cannot be the sides of a triangles, because $2 + 4 < 7$. The condition for forming a triangle is the sum of the lengths of any two sides of a triangle is greater than the length of the third side.

(iii) **Define proportion.**

Ans Equality of two ratios is defined as the proportion.

If $a : b = c : d$, then a, b, c and d are said to be a proportion.

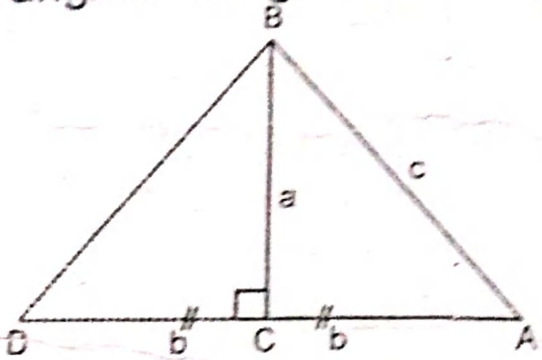
(iv) **Define similar triangles.**

Ans Two (or more) triangles are called similar, if they are equiangular and measure of their corresponding sides are proportional. The symbol for similar triangle is (~).

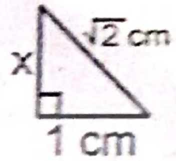
(v) **What is meant by converse of Pythagoras theorem?**

Ans Converse of Pythagoras theorem is:

If the square of one side of a triangle is equal to the sum of the squares of the other two sides, then the triangle is a right angled triangle.



(vi) **Find the value of unknown x:**



Ans In right angled ΔABC is:

$(m\,AC)^2 = (m\,AB)^2 + (m\,BC)^2$ (Pythagoras Theorem)

By putting values:

$$(\sqrt{2})^2 = (x)^2 + (1)^2$$
$$2 = x^2 + 1$$
$$x^2 = 2 - 1$$
$$x^2 = 1$$

By taking square root:

$$\sqrt{x^2} = \sqrt{1}$$
$$x = 1 \text{ cm}$$

(vii) **Define triangular region.**

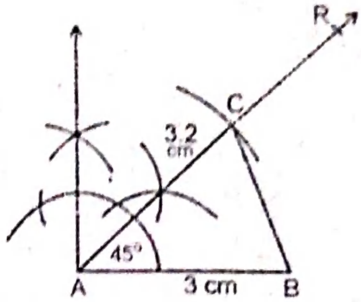
Ans A triangular region is the union of a triangle and its interior, *i.e.*, the three line segments forming the triangle and its interior.

(viii) **Define circumcentre of a triangle.**

Ans The point of concurrency of the perpendicular bisectors of the sides of a triangle is called its circumcentre.

(ix) **Construct Δ ABC, in which : m $\overline{AB}$ = 3 cm, m $\overline{AC}$ = 3 cm, m$\angle$ A = 45°**

Ans



Steps of Construction:

) Take a line segment $\overline{AB} = 3$ cm.

i) Make an angle of 45° at A.

ii) Take A as centre and cut off $\overline{AC} = 3.2$ cm on $\overrightarrow{AR}$.

v) Join C to B.

(v) ABC is the required triangle.

(Part-II)

IOTE: Attempt Three (3) questions in all. But question No. 9 is Compulsory.

Q.5.(a) Solve with help of Cramer's rule: **(4)**

$$4x + 2y = 8$$

$$3x - y = -1$$

Ans Write the equations in matrix form

$$\begin{bmatrix} 4 & 2 \\ 3 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ -1 \end{bmatrix}$$

Let, $AX = B$

Where,

$$A = \begin{bmatrix} 4 & 2 \\ 3 & -1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}, B = \begin{bmatrix} 8 \\ -1 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 4 & 2 \\ 3 & -1 \end{vmatrix}$$

$$|A| = (4)(-1) - (2)(3)$$

$$|A| = -4 - 6$$

$$\boxed{|A| = -10}$$

$$|A| = \begin{vmatrix} 8 & 2 \\ -1 & -1 \end{vmatrix}$$

$$|A_x| = (8)(-1) - (-1)(2)$$
$$|A_x| = -8 + 2$$
$$\boxed{|A_x| = -6}$$
$$|A_y| = \begin{vmatrix} 4 & 8 \\ 3 & -1 \end{vmatrix}$$
$$|A_y| = (4)(-1) - (3)(8)$$
$$|A_y| = -4 - 24$$
$$\boxed{|A_y| = -28}$$

So, $x = \dfrac{|A_x|}{|A|} = \dfrac{-6}{-10} = \dfrac{3}{5}$

$y = \dfrac{|A_y|}{|A|} = \dfrac{-28}{-10} = \dfrac{14}{5}$

Solution Set $= \left[\dfrac{3}{5}, \dfrac{14}{5}\right]$

(b) Solve the equation for real x and y

$(3 + 4i)^2 - 2(x - yi) = x + yi$

Ans $(12)^2$ 24 12

$$(3 + 4i)^2 - 2(x - yi) = x + yi$$
$$9 + 24i + 16i^2 - 2x - 2yi = x + yi$$
$$9 + 24i + 16(-1) - 2x + 2yi = x + yi$$
$$9 + 24i - 16 - 2x - x + 2yi - yi = 0$$
$$9 + 24i - 16 - 3x + yi = 0$$
$$-3x + yi = 16 - 9 - 24i$$
$$-3x + yi = 7 - 24i$$

By comparing w.r.t $a + bi$ form:

$$-3x = 7 \quad \ldots (i)$$

and

$$y = -24 \quad \ldots (ii)$$

from (i) $-3x = 7$

$$x = \frac{-7}{3}$$

$\therefore$ $\boxed{x = \dfrac{-7}{3}}, \boxed{y = -24}$

Q.6.(a) Use log tables to find the value of: **(4)**

$$\frac{(438)^3\sqrt{0.056}}{(388)^4}$$

Ans Let $x = \dfrac{(438)^3\sqrt{0.056}}{(388)^4}$

$$x = \frac{(438)^3(0.056)^{1/2}}{(388)^4}$$

By taking log, both sides:

$$\log x = \log\left[\frac{(438)^3(0.056)^{1/2}}{(388)^4}\right]$$

$$\log x = 3\log 438 + \frac{1}{2}\log 0.056 - 4\log 388$$

$$\log x = 3(2.6415) + \frac{1}{2}(\bar{2}\,.\,7482) - 4(2.5888)$$

$$\log x = 7.9245 + \bar{1}\,.\,3741 - 10.3552$$

$$\log x = 7.9245 - 1 + 0.3741 - 10.3552$$

$$\log x = -3.0566$$

$$\log x = -3.0566 - 1 + 1$$

$$\log x = \bar{4}\,.\,9434$$

Taking antilog both sides,

$$x = \text{Antilog}\,(\bar{4}\,.\,9434)$$

$$\boxed{x = 0.0008778}$$

(b) Simplify: $\dfrac{\sqrt{a^2+2}+\sqrt{a^2-2}}{\sqrt{a^2+2}-\sqrt{a^2-2}}$ **(4)**

Ans With Rationalization:

$$= \frac{\sqrt{a^2+2}+\sqrt{a^2-2}}{\sqrt{a^2+2}-\sqrt{a^2-2}} \times \frac{\sqrt{a^2+2}+\sqrt{a^2-2}}{\sqrt{a^2+2}+\sqrt{a^2-2}}$$

$$= \frac{(\sqrt{a^2+2}+\sqrt{a^2-2})^2}{(\sqrt{a^2+2})^2-(\sqrt{a^2-2})^2}$$

$$= \frac{(\sqrt{a^2+2})^2+(\sqrt{a^2-2})^2+2(\sqrt{a^2+2})(\sqrt{a^2-2})}{(a^2+2)-(a^2-2)}$$

$$= \frac{(a^2+2)+(a^2-2)+2\sqrt{(a^2+2)(a^2-2)}}{a^2+2-a^2+2}$$

$$= \frac{a^2+2+a^2-2+2\sqrt{(a^2)^2-(2)^2}}{4}$$

$$= \frac{2a^2+2\sqrt{a^4-4}}{4}$$

$$= \frac{2(a^2+\sqrt{a^4-4})}{4}$$

$$= \frac{a^2+\sqrt{a^4-4}}{2}$$

Q.7.(a) Factorize: $2x^3 + x^2 - 2x - 1$

Ans We have; $P(x) = 2x^3 + x^2 - 2x - 1$

The possible factors of the constant term

$P = -1$ are $\pm 1, \pm \frac{1}{2}$.

From factors of the constant

Let, $x = -1$

$$\begin{aligned} P(-1) &= 2(-1)^3 + (-1)^2 - 2(-1) - 1 \\ &= 2(-1) + 1 + 2 - 1 \\ &= -2 + 3 - 1 \\ &= 0 \end{aligned}$$

Hence, $x = -1$ is a zero of $P(x)$;

As,
$$x - a = 0$$
$$x - (-1) = 0$$
$$x + 1 = 0$$

So, $x + 1$ is the 1^{st} factor of $P(x)$.

Similarly,

Let $x = 1$

$$\begin{aligned} P(1) &= 2(1)^3 + (1)^2 - 2(1) - 1 \\ &= 2(1) + 1 - 2 - 1 \\ &= 2 + 1 - 2 - 1 \\ &= 0 \end{aligned}$$

Hence, $x = 1$ is a zero of $P(x)$;

As, $x - a = 0$

$$x - 1 = 0$$

So, $x - 1$ is the 2^{nd} factor of P(x).

Again;

Let $x = \frac{-1}{2}$

$$P(\tfrac{-1}{2}) = 2(\tfrac{-1}{2})^3 + (\tfrac{-1}{2})^2 - 2(\tfrac{-1}{2}) - 1$$

$$= 2(\tfrac{-1}{8}) + (\tfrac{1}{4}) + 1 - 1$$

$$= \frac{-1}{4} + \frac{1}{4} + 0$$

$$= 0$$

Hence, $x = \frac{-1}{2}$ is a zero of P(x)

As, $x - a = 0$

$$x - (\tfrac{-1}{2}) = 0$$

$$x + \frac{1}{2} = 0$$

$$(2)(x) + (2)(\tfrac{1}{2}) = (0)(2)$$

$$2x + 1 = 0$$

So, $2x + 1$ is the last factor of P(x). As from the expression, there exist maximum three factors, *i.e.*,

The factors of P(x) are $= (x - 1)(x + 1)(2x + 1)$

(b) Find the H.C.F of the following polynomials: (4)

$x^2 - 4$, $x^2 + 4x + 4$, $2x^2 + x - 6$

Ans By factorization:

$$x^2 - 4 = (x + 2)(x - 2)$$

$$x^2 + 4x + 4 = (x + 2)^2 = (x + 2)(x + 2)$$

$$2x^2 + x - 6 = 2x^2 + 4x - 3x - 6$$

$$= 2x(x + 2) - 3(x + 2)$$

$$= (x + 2)(2x - 3)$$

Hence, H.C.F $= x + 2$

Q.8.(a) Find the solution set of the equation:

$$|8x - 3| = |4x + 5|$$
$$8x - 3 = \pm(4x + 5)$$

Ans Since two numbers having the same absolute value are either equal or differ in sign, therefore, the given equation is equivalent to

$8x - 3 = 4x + 5$	$8x - 3 = -(4x + 5)$
$8x - 4x = 5 + 3$	$8x - 3 = -4x - 5$
$4x = 8$	$8x + 4x = -5 + 3$
$\boxed{x = 2}$	$12x = -2$
	$\boxed{x = \frac{-1}{6}}$

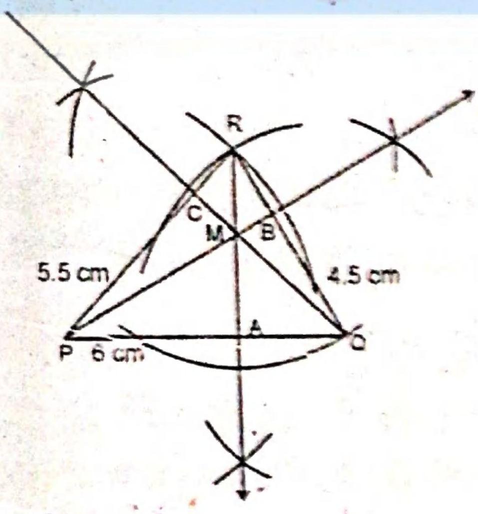
On checking, we find that $x = 2, x = \frac{-1}{6}$ both satisfy the original equation.

Hence the solution set $= \left\{\frac{-1}{6}, 2\right\}$.

(b) **Construct ΔPQR, draw its altitudes and show that they are concurrent:**

$m\overline{PQ} = 6$ cm, $m\overline{QR} = 4.5$ cm and $m\overline{PR} = 5.5$ cm

Ans



Steps of Construction:

(i) Take a line segment $\overline{PQ} = 6$ cm.

(ii) Take P as centre and draw an arc of 5.5 radius.

(iii) Take Q as centre and draw an arc of 4.5 radius.

iv) Join R to P and Q.

$\because$ PQR is the required triangle.

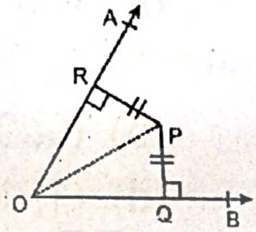
v) Drop $\overline{RA}$, $\overline{PB}$ and $\overline{QC}$ perpendiculars to $\overline{PQ}$, $\overline{QR}$ and $\overline{RP}$, respectively.

$\overline{RA}$, $\overline{PB}$ and $\overline{QC}$ are concurrent at M.

Q.9. Prove that any point inside an angle, equidistant from its arms, is on the bisector of it. (8)

Ans **Given:**

Any point P lies inside $\angle AOB$ such that $\overline{PQ} \cong \overline{PR}$, where $\overline{PQ} \perp \overline{OB}$ and $\overline{PR} \perp \overline{OA}$.



To Prove:

Point P is on the bisector of $\angle AOB$.

Construction:

Join P to O.

Proof:

Statements	Reasons
In $\Delta POQ \leftrightarrow \Delta POR$	
$\angle PQO \cong \angle PRO$	given (right angles)
$\overline{PO} \cong \overline{PO}$	common
$\overline{PQ} \cong \overline{PR}$	given
$\therefore$ $\Delta POQ \cong \Delta POR$	H.S $\cong$ H.S
Hence, $\angle POQ \cong \angle POR$	(corresponding angles of congruent triangles)
i.e., P is on the bisector of $\angle AOB$.	

OR

Prove that triangles on the same base and of the same (*i.e.,* equal) altitudes are equal in area.

Ans For Answer see Paper 2014 (Group-I), Q.9.(OR).